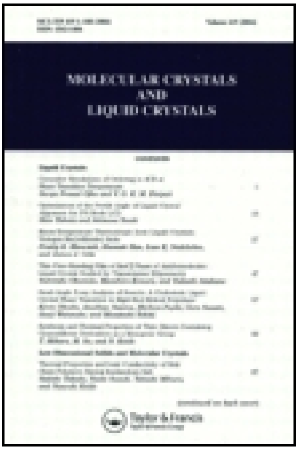

Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl16

The Orientational Optical Non-Linearity of Liquid Crystals

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Version of record first published: 20 Apr 2011.

Mol. Cryst. Liq. Cryst., 1981, Vol. 69, pp. 19–30
0026-8941/81/6902-0019$06.50/0

The Orientational Optical Non-Linearity of Liquid Crystals

II. Cholesterics

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(Received, March 17, 1980; in final form, July 25, 1980)

The cubic optical non-linearity of a cholesteric liquid crystal (CLC) having a planar helical texture is considered. A study was made of an effect which has a minimal establishment time and is due to the change in the director distribution along the helix under the orientational influence of the light field. Polarisation peculiarities of the non-linearity, when the light wave propagates along the helical axis, are discussed. Self-focusing should not occur for circulary polarised light, but has the constant $\varepsilon_2 \sim 8 \cdot 10^{-8}$ cm^3/erg for linearly polarised light. The additional rotation of the axes of elliptical polarisation is predicted to be proportional to the light intensity and determined by the same constant ε_2; this rotation is identical for both right- and left-handed CLC. Non-linear optical activity is absent in the approximation under consideration.

Analogous effects for light propagating through an oriented nematic are discussed.

1 INTRODUCTION

In the last few years, the non-linear optics of liquid crystals has attracted great attention[1] both from theorists and experimentalists. The cubic non-linear effects conditioned by the thermal mechanism and the orientational non-linearity of the isotropic phase near to the nematic-isotropic transition point have been investigated in detail.[3,4] The anomalously large optical non-linearity of a NLC mesophase conditioned by director reorientation under the action of light fields has also been discussed.[5]

Light propagation in a CLC is considered theoretically in the present paper. As a non-linearity mechanism, we consider the most strong and rapid effect, i.e., the director reorientation inside the unchanged periodic structure of the CLC. Calculation gives the non-linear constant a value of $\varepsilon_2 \sim 8 \cdot 10^{-8}$ cm^3/erg which is greater than that for the optical non-linearity of CS_2 by 3 orders. Polarisation peculiarities of non-linear effects in a CLC are discussed in detail.

2 FREE ENERGY AND EQUATIONS FOR THE DIRECTOR AND LIGHT FIELD

For the free energy of a unit volume of a CLC which is under the action of a light field, we use the form

$$F = \tfrac{1}{2}[k_{11}(\operatorname{div}\mathbf{n})^2 + k_{22}(\mathbf{n}\operatorname{rot}\mathbf{n} + q)^2 + k_{33}(\mathbf{n}\times\operatorname{rot}\mathbf{n})^2 - \frac{\varepsilon_a}{8\pi}(\mathbf{nE})(\mathbf{nE}^*)]. \quad (1)$$

Here $\hat{\mathbf{n}}$ is the unit vector determining the local direction of the mean molecular orientation (the director). If the z-axis of a cartesian coordinate system is directed along the cholesteric helix axis, then $\mathbf{n}$ takes the form

$$\mathbf{n} = (n_x, n_y, 0) = \{\cos\theta(z), \sin\theta(z), 0\}.$$

The dependence $\theta(z) = qz$ minimises the free energy (1) in the absence of a light field $\mathbf{E}$ for the undistorted helical structure.

Here $q = 2\pi/P$ is the wave vector of the helix with pitch P. Also the following notations are introduced in Eq. (1): k_{11}, k_{22}, k_{33} (din), the Frank constants; $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp$, the anisotropy of the dielectric susceptibility of the mesophase; ω the light frequency. The complex amplitude $\mathbf{E}(\mathbf{r}, t)$ of the quazi-monochromatic light field is connected with the real vector $\mathbf{E}_{\text{real}}(\mathbf{r}, t)$ by the relation

$$\mathbf{E}_{\text{real}}(\mathbf{r}, t) = 0.5[\mathbf{E}(\mathbf{r}, t)\exp\{-i\omega t\} + \mathbf{E}^*(\mathbf{r}, t)\exp\{i\omega t\}].$$

In Eq. (1) we have omitted terms not depending on the director orientation $\mathbf{n}$. Besides that, after substitution of the real light field $\mathbf{E}_{\text{real}}$ into the expression for the free energy, we omit the terms

$$\sim EE\exp\{-i2\omega t\} + E^*E^*\exp\{i2\omega t\}.$$

Since typically $\omega/2\pi \sim 10^{15}$ Hertz, and such rapidly oscillating terms with doubled light frequency 2ω do not influence the macroscopic movement of the director.

Consider the case of a light wave propagating along the helix axis. Let us introduce the circular polarisation unit vectors

$$\mathbf{e}_R = \frac{\mathbf{e}_x + i\mathbf{e}_y}{\sqrt{2}}, \mathbf{e}_L = \frac{\mathbf{e}_x - i\mathbf{e}_y}{\sqrt{2}}$$

$\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ form a right-handed orthogonal system of unit vectors, and $\mathbf{e}_z$ is directed along the cholesteric helix axis. Then the wave field will be written in the form $\mathbf{E} = \mathbf{e}_R E_R + \mathbf{e}_L E_L$, where $E_R(z)$ and $E_L(z)$ are the amplitudes of the right and left polarised wave, respectively.

The Maxwell equations for E_R and E_L have the form (see, for example):[6]

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} + k^2\right)E_L = -\frac{\omega^2}{c^2}\frac{\varepsilon_a}{2}\exp\{i2\theta(z)\}E_R, \tag{2a}$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} + k^2\right)E_R = -\frac{\omega^2}{c^2}\frac{\varepsilon_a}{2}\exp\{-i2\theta(z)\}E_L \tag{2b}$$

where $k^2 = (\omega/c)^2\varepsilon_\parallel + \varepsilon_\perp/2 = (\omega/c)^2\varepsilon_0$. This system has an exact solution for the simple case of linear dependence of $\theta(z)$ on z, $\theta(z) = qz$, see:[6]

$$\begin{aligned} E_L(z) &= c_1(z)a_1 e^{i(p_1+q)z} + c_2(z)a_2 e^{i(p_2+q)z}, \\ E_R(z) &= c_1(z)b_1 e^{i(p_1-q)z} + c_2(z)b_2 e^{i(p_2-q)z} \end{aligned} \tag{3}$$

where $c_1(z) = \text{const}$, $c_2(z) = \text{const}$, and $a_{1,2}$ and $b_{1,2}$ are real quantities to be determined from the equation

$$[(p_i + q)^2 - k^2]a_i - \left(\frac{\omega}{c}\right)^2\frac{\varepsilon_a}{2}b_i = 0$$

Parameters $p_{1,2}$ are those two roots (out of four) from the equation

$$(p^2 + q^2 - k^2)^2 - 4q^2p^2 - \left(\frac{\omega}{c}\right)^4\left(\frac{\varepsilon_a}{2}\right)^2 = 0$$

which correspond to light propagation along the $+z$ direction. By this we assume that the frequency ω of the incident light is outside the frequency range of Bragg reflection.

We pick out specially amplitudes $c_1(z)$ and $c_2(z)$, having in mind that with regard to self-action these amplitudes will vary slowly along z axis. However, in linear optics, i.e., when $\theta = qz$, the values of c_1 and c_2 are strictly constant.

The free energy expressed through the amplitudes of circulary polarised waves takes the form

$$F = \tfrac{1}{2}k_{22}\left(\frac{\mathrm{d}\theta}{\mathrm{d}z} - q\right)^2 - \frac{\varepsilon_a}{32\pi}\{|E_r|^2 + |E_L|^2 + E_L E_R^* e^{-i2\theta(z)} + E_L^* E_R e^{i2\theta(z)}\} \tag{4}$$

Since our purpose is to consider stationary processes, we must solve the Eqs. (2) together with the equation determining the equilibrium distribution of the director field in the CLC under the action of the external light field. The equation for $\theta(z)$ is obtained from the Lagrange variational equation, where $\theta(z)$ appears as an independent variable function:

$$\frac{\delta F}{\delta \theta} - \frac{\partial}{\partial x_i} \frac{\delta F}{\delta(\partial\theta/\partial x_i)} = 0 \tag{5}$$

Inserting Eq. (4) into Eq. (5) we obtain

$$k_{22} \frac{d^2\theta}{dz^2} + i \frac{\varepsilon_a}{16\pi} \{E_R E_L^* e^{i2\theta(z)} - E_R^* E_L e^{-i2\theta(z)}\} = 0. \tag{6}$$

3 DIRECTOR REORIENTATION IN THE FIRST ORDER OF LIGHT WAVE POWER

The interaction of the strong electromagnetic field with the medium results in some distortion of the helical structure. We shall consider this circumstance by writing $\theta(z)$ as $\theta(z) = qz + \alpha(z)$, where $\alpha(z)$ is the small disturbance in the helical phase determined by the wave field.

The solution of the problem will be sought in the form (3) with amplitudes $c_1(z)$ and $c_2(z)$ slowly dependent on z. To be more accurate, the rapidly varying small terms δE_R and δE_L should also be added to Eq. (3). But those terms, in the first non-vanishing order for the light wave intensity, do not add to the changes in the basic amplitudes $c_1(z)$ and $c_2(z)$, and therefore we do not consider them explicitly. Then Eq. (6) can be approximately integrated:

$$\alpha(z) = i\kappa\{c_1 c_2^* e^{i(p_1 - p_2)z} - c_1^* c_2 e^{-i(p_1 - p_2)z}\} + \beta z + \phi_0 \tag{7}$$

where

$$\kappa = \frac{\varepsilon_a(a_2 b_1 - a_1 b_2)}{16\pi k_{22}(p_1 - p_2)^2}.$$

The definition of constants β and ϕ_0 in Eq. (7) requires additional information (in comparison with Eq. 6); we mean boundary conditions and free energy minimization. Let us explain this through the following considerations. Equation (6) deals with the local equilibrium of the CLC helix. Therefore, even in the absence of fields, $E_L = E_R = 0$, the general solution has the form $\alpha(z) = \beta z + \phi_0$. Only the search for the free energy minimum gives the value $\beta = 0$. The constant ϕ_0 is determined by the condition on the director at $z = 0$ (the medium boundary).

In the presence of light fields, the pitch of the helical structure can be, generally speaking, changed; this corresponds to $\beta \neq 0$, but for more details see.[7] However, the change in pitch over a space corresponding to many periods requires, obviously, a much longer time for its establishment;

it also depends on boundary conditions and the properties of the beam in the transverse direction.

Below we first restrict ourselves to effects which are established very rapidly; we will consider the condition of pitch conservation at the moment of field switching, $\beta = 0$, and take into account the profile distortions $\theta(z) = qz + \alpha(z)$ of the CLC helix, when the helix period is fixed. As will be seen below, the final results for E_R and E_L are independent of the integer constant ϕ_0 from Eq. (7).

4 SOLUTIONS OF EQUATIONS FOR LIGHT FIELDS

Assuming that $|\alpha(z)| \ll 1$ we present the exponential multipliers of Eq. (2) in the form

$$\exp\{\pm i2\theta(z)\} \approx (1 \pm i2\alpha(z)) \exp\{\pm i2qz\}.$$

Then from Eqs. (3) and (2a) one obtains

$$a_1 \frac{d^2c_1}{dz^2} + a_1(p_1 + q)\frac{dc_1}{dz} e^{i(p_1+q)z} + a_2 \frac{d^2c_2}{dz^2} + a_2(p_2 + q)\frac{dc_2}{dz} e^{i(p_2+q)z}$$

$$= -i\kappa\left(\frac{\omega}{c}\right)^2 \frac{\varepsilon_q}{2} \{b_1 c_1^2 c_2^* e^{i(2p_1 - p_2 + q)z} - b_1 |x_1|^2 c_2 e^{i(p_2+q)z}$$

$$+ b_2 |c_2|^2 c_1 e^{i(p_1+q)z} - b_2 c_2^2 c_1^* e^{i(2p_2 - p_1 + q)z}\} \tag{8a}$$

and the analogous Eqs. (8b) which follows from (3) and (2b); this we do not write down explicitly. Below, we neglect in Eq. (8) the terms $\sim d^2c_{1,2}/dz^2$, which are small in comparison with the terms $k\, dc_{1,2}/dz$. Equation (8a) and the second (unwritten) Eq. (8b) allows us then to determine the values of $dc_{1,2}/dz$. For example

$$\frac{dc_1}{dz} = iM_1[c_1^2 c_2^* e^{i(p_1 - p_2)z} - |c_1|^2 c_2 e^{-i(p_1 - p_2)z}]$$

$$+ iM_2[|c_2|^2 c_1 - c_2^2 c_1^* e^{-i2(p_1 - p_2)z}] \tag{9}$$

where

$$M_1 = \left(\frac{\omega}{c}\right)^2 \cdot \frac{\varepsilon_a^2(a_2 b_1 - a_1 b_2)}{32\pi k_{22}(p_1 - p_2)^2}$$

$$\cdot \frac{a_1 a_2(p_2 + q) + b_1 b_2(p_2 - q)}{a_2 b_1(p_1 - q)(p_2 + q) - a_1 b_2(p_1 + q)(p_2 - q)},$$

$$M_2 = \left(\frac{\omega}{c}\right)^2 \frac{\varepsilon_a^2(a_2 b_1 - a_1 b_2)}{32\pi k_{22}(p_1 - p_2)^2} \cdot \frac{a_2^2(p_2 + q) + b_2^2(p_2 - q)}{a_2 b_1(p_1 - q)(p_2 + q) - a_1 b_2(p_1 + q)(p_2 - q)}$$

and the analogous equation for dc_2/dz with the constants $\tilde{M}_1$ and $\tilde{M}_2$.

Since we assumed $c_{1,2}(z)$ to be slowly varying functions of z, all the exponentially oscillating terms in the RHS of Eq. (9) should be omitted. In a more accurate sense, such oscillating terms give rise to just those small, rapidly varying terms δE_R and δE_L, which are not of interest to us. Hence, we obtain the system of equations for *slowly* varying amplitudes

$$\frac{dc_1}{dz} = iM_2 |c_2|^2 c_1$$

$$\frac{dc_2}{dz} = i\tilde{M}_2 |c_1|^2 c_2 \tag{10}$$

The solution of those equations is obvious. The intensities $|c_1|^2$ and $|c_2|^2$ do not depend on z. The change δk_1 of the wave vector of the first wave is proportional to the intensity $|c_2|^2$ of the second wave, and *vice versa.*

The initial values of $c_1(z = 0)$ and $c_2(z = 0)$ are determined by the boundary conditions. For simplification of estimations, let us consider the case $\varepsilon_a \lesssim 1$.

Then $a_1/b_1 \sim \varepsilon_a$, $b_2/a_2 \sim \varepsilon_a$. Neglecting the terms $\sim\varepsilon_a^3$, one obtains

$$\mathbf{E}(z) = \mathbf{e}_R b_1 c_1(z) e^{i(p_1 - q)z} + \mathbf{e}_L a_2 c_2(z) e^{i(p_2 + q)z}$$

$$\equiv \mathbf{e}_R E_R(z) + \mathbf{e}_L E_L(z), \tag{11}$$

$$E_R(z) = E_R(0) \exp\{i2\eta |E_L|^2 z + i(k + \rho + \phi)z\},$$

$$E_L(z) = E_L(0) \exp\{i2\eta |E_R|^2 z + i(k + \rho - \phi)z\} \tag{12}$$

where

$$\eta = \frac{\omega}{c} \frac{\varepsilon_a^2}{256\pi\sqrt{\varepsilon_0} k_{22} q^2}, \quad \rho = \left(\frac{\omega}{c}\right)^4 \frac{\varepsilon_a^2}{32k(q^2 - k^2)},$$

$$\phi = \left(\frac{\omega}{c}\right)^4 \frac{\varepsilon_a^2}{32q(k^2 - q^2)}.$$

5 POLARISATION PECULIARITIES OF OPTICAL NON-LINEARITY FOR A CLC

5a Elliptical rotation of polarisation and the absence of non-linear optical activity

At the point of entry of the medium where $z = 0$, an elliptically polarised wave be given:

$$\mathbf{E} = E(\mathbf{e}_x \cos v + i\mathbf{e}_y \sin v) \tag{13a}$$

Then splitting these fields into E_R and E_L components and inserting them into Eq. (11) gives

$$\mathbf{E}(z) = [\cos v(\mathbf{e}_x \cos \Lambda z - \mathbf{e}_y \sin \Lambda z) + i \sin v(\mathbf{e}_y \cos \Lambda z + \mathbf{e}_x \sin \Lambda z)]. \cdot \exp\{i(k + \rho)z + i\eta |E|^2 z\}E, \tag{13b}$$

$$\Lambda = \phi - \eta |E|^2 \sin 2v. \tag{13c}$$

Equation (13) corresponds to the following physical effects.

1) The elliptical polarisation semi-axis relation ($b/a = \tan v$) is preserved during the propagation process.

2) There is an elliptical polarisation rotation with speed ϕ (rad/cm) typical of CLC linear optics. The sign of this rotation changes when a right-handed CLC is substituted for a left-handed CLC.

3) There is an additional rotation of ellipse orientation with speed $\eta |E|^2 \sin 2v$, proportional to the incident light intensity. The magnitude and the sign of this rotation are defined by the rate of the circularity $\sin 2v$ which achieves its maximal value when $|\sin 2v| \to 1$. But in this case, the wave itself becomes either right circular or left circular, and the notion of orientation of the semi-major axis of the ellipse gradually loses its sense. A slow non-linear rotation of the orientation of the ellipse during the propagation time is opposite to the direction of rotation of the end of the vector of the elliptically polarised wave, which rotates at light frequencies. This effect is analogous to the rotation of the polarisation ellipse in the isotropic non-linear medium.[8] Note that the self-rotation sign of the elliptical polarisation is independent of the CLC sign. It can be said that non-linear optical activity in its usual sense is absent in our approximation for a CLC with constant pitch.

4) There is a total shift of the light field phase proportional to the wave intensity. For the wave with polarisation close to circular ($|\cos v| \approx |\sin v|$),

such a phase shift disappears. It is more convenient to follow this, not by Eq. (13), but by a more simple formula (12). Really, when $|L| \to 0$, $|R| =$ const, only the L-wave, whose amplitude is negligible, has non-linear addition to the phase, while the phase of the basic R-wave changes a little, by the quantity $2i\eta|L|^2 z \to 0$.

Let us make numerical estimations for the self-rotation of the polarisation ellipse. Let the wavelength in a vacuum be $\lambda = 0.5\ \mu$m; $\sqrt{\varepsilon_0} = 1.3$; $\varepsilon_a \sim 0.1$; the cholesteric pitch $P = 6 \cdot 10^{-4}$ cm ($q = 10^4$ cm^{-1}); the Frank constant $k_{22} \sim 10^{-6}$ dyn. Let us write the rate of self-rotation of the ellipse axis $d\phi_e/dz$ in the form

$$\frac{d\phi_e}{dz} = \eta|E|^2 \sin 2\nu = \tilde{\eta}\,\frac{cn|E|^2}{8\pi} \sin 2\nu \tag{14}$$

where η is defined by Eq. (11b). Then, for the selected values, from Eqs. (14) and (11b) we obtain $\eta \sim 1{,}1 \cdot 10^{-2}$ cm^2/erg. If we express the Pointing vector $S = cn|E|^2/8\pi$ in watts/cm^2, then the constant $\tilde{\eta} \approx 7.5 \cdot 10^{-5}$ cm/watt. From this it is seen that if the power density is $S \approx 10^6$ watt/cm^2, the self-rotation of the ellipse axis at the length $l \sim 10^{-2}$ cm is $\phi_e \sim 1$ rad. The establishment time of the non-linearity connected with the spatially inhomogeneous reorientation of the director is $\tau \sim \gamma/k_{22}(2q)^2 \sim 10^{-4} s$ for the viscosity constant $\gamma \sim 4 \cdot 10^{-2}$ poise (compare with Refs. 5, 10). At the absorption coefficient $\sim$0.1 cm^{-1}, the energy release from the incident beam during the time $\tau \sim 10^{-4}$ makes $\sim$1 j/cm^3, which for a heat capacity of 0.5 j/cm^3 deg gives a heating of $\sim$0.5°C. It is hoped that such a heating will not prevent us from observing the self-rotation of the ellipse axis which is of interest to us. We would emphasise that the sign of this effect depends on the sign of the elliptical circularity, but not on the helix sign.

5 LIGHT SELF-FOCUSING BY THE ORIENTED CLC

The presence of a phase shift proportional to the intensity testifies to the possibility of a light self-focusing effect. Let us remember that the self-focusing effect vanishes for circulary polarized light. Therefore, we shall consider the case for linear polarised incident light, $\nu = 0$, in obtaining numerical estimations. For the linear polarised wave, the shift rotation of the polarisation, in accordance with Eq. (13), is intensity independent. The common phase, non-linear shift for $\nu = 0$ is given by the multiplier

$$E \sim \exp\{i\eta|E|^2 z\}$$

i.e., it numerically coincides with the rate of rotation of the ellipse axis when $\nu = \pi/4$ (compare with Eq. 14). Thus, for the same parameters of the medium and the beam in the case of a linear polarised entrant wave with diffractional angular divergence, the self-focusing effect will bring about a change in the emergent beam divergence by about 100%. The self-focusing critical power[11,12] amounts to

$$W_c = \frac{\lambda^2 cn}{32\pi\varepsilon_2} = \frac{\lambda cn}{16\eta}$$

and for $\eta = 1.1 \cdot 10^{-2}$ cm^2/erg we have $W_c = 1.1$ watt. In the case of $W/W_c \sim 10^6$ and a beam transverse size $a \sim 10^{-2}$ cm, the intrinsic self-focusing length $l = ka^2(W/W_c)^{-1/2}$ (see Ref. 13), making $l \sim 1.5 \cdot 10^{-2}$ cm.

6 SELF-ACTION EFFECTS IN THE ORIENTED NEMATIC MESOPHASE

The strongest effects of light self-focusing in an oriented NLC have been considered in our paper.[5] There it has been shown that those effects appear only when the incident wave is extraordinary and falls at an oblique angle to the director. However, the non-linearity establishment time proved to be very large ($\tau \sim 0, 1 - 1$ s). Here we want to discuss the non-linear effects which arise when the wave falls normally to the planar oriented NLC layer. We assume that the undisturbed direction of the NLC director is supported by the external magnetic field **H** directed along the y-axis (compare with Ref. 5), and that the z-axis is normal to the NLC layer boundary. Then by introducing notations $\varepsilon_0 = (\varepsilon_\parallel + \varepsilon_\perp)/2$, $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp$, $k_0 = \omega\sqrt{\varepsilon_\perp}/c$, $k_2 = \omega\sqrt{\varepsilon_\parallel}/c$, $q = K_e - k_0 > 0$, $\kappa_a = \kappa_\parallel - \kappa_\perp$,

$$\mathbf{E}(z) = \mathbf{e}_y E_y(z) \exp\{ik_e z\} + \mathbf{e}_x E_x(z) \exp\{ik_0 z\}$$

we obtain the system of equations

$$\frac{dE_x}{dz} = i2\eta_n |E_y|^2 E_x, \tag{15a}$$

$$\frac{dE_y}{dz} = i2\eta_n |E_x|^2 E_y, \tag{15b}$$

$$\eta_n = \frac{\omega}{c\sqrt{\varepsilon_0}} \frac{\varepsilon_a^2}{64\pi} \frac{1}{k_{22}q^2 + \kappa_a H^2}. \tag{15c}$$

To obtain this system we have taken into account the following effects. In the zero approximation for the light field intensity, the amplitudes $E_x(z)$ and $E_y(z)$ are constant. Its interference $E_x E_y^* \exp\{-iqz\}$ and $E_x^* E_y \exp\{iqz\}$ contributes to the forces causing the director reorientation. The calculation of the first-order reorientation under the action of a spatially inhomogeneous force has been given in our paper.[5] The scattering of the E_x and E_y fields on the resulting periodic lattice of the dielectric susceptibility gives just the non-linear terms in the right-hand side of Eqs. (15a, b).

It is not difficult to see that both the physical mechanism and the structure of Eq. (15) for the NLC are quite similar to the mechanism and Eq. (11) for the CLC with the replacement of R and L (i.e., the CLC eigenvalues) by o and e (i.e., the NLC eigenvalues).

The NLC non-linearity vanishes when the incident wave has any of the eigen-polarisation $\mathbf{E} \sim \mathbf{e}_x$ or $\mathbf{E} \sim \mathbf{e}_y$. For the arbitrary elliptical wave, the ellipse parameters are changed even at small light intensities by the laws typical for an anisotropic medium. The allowance for non-linearity results, first of all, in the phase common shift (the self-focusing effect), which is maximal for the case $|E_x| = |E_y|$. This corresponds to the elliptically polarised wave with the ellipse axis at 45° to x, y. The circulary polarised waves, and also the waves with linear polarisation at 45° to x, y are the particular cases. Besides, a self-birefringence effect should take place analogous to the self-rotation of the polarisation of the ellipse axis.

Since one may expect a larger value of ε_a for the nematic liquid crystal (for example $\varepsilon_a \approx 1$ at $T = 125°C$ for Paa),[6] then the numerical estimations obtained prove to be very favourable; $W_c \sim 10^{-3}$ watt, $\tau \sim 10^{-4}$ s.

Thus, one can hope to observe relatively rapid (10^{-4} s) light self-focusing in a NLC when the wave falls normally to the planar oriented NLC layer, if the incident wave polarisation makes an oblique angle with the director.

7 CONCLUSION

Thus, fairly strong and relatively rapid effects of light wave self-action in an oriented NLC and CLC are predicted and calculated. In both cases the mechanism is connected with the spatially inhomgeneous reorientation of the director. The non-linearity must appear in the form of self-focusing and self-rotation of the axis of elliptical polarisation for a CLC, and for a NLC, in the form of self-focusing and self-birefringence. The numerical estimations show that the predicted effects are capable of observation.

Acknowledgements

The authors gratefully acknowledge S. M. Arakelian, N. B. Baranova, L. M. Blinov, E. I. Kats, V. V. Shkunov, and Yu. S. Tchilingarian for many useful discussions.

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